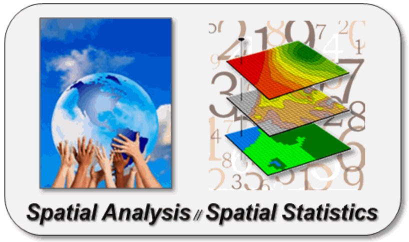
*Spatial*STEM:

A Mathematical/Statistical Framework for Understanding and Communicating Map Analysis and Modeling



Part 2) **Spatial Analysis**. Modern digital maps are "numbers first, pictures later." In map-*ematical* processing, these data can be conceptualized as a set of "**floating maps**" with a common registration that enables the computer to **"look" down and across the stack of map layers** to spear or corral sets of numbers for processing. *Spatial Analysis* involves quantitative analysis of the "**spatial context**" of mapped data, such as add, subtract, multiply, divide, exponentiation, root, log, cosine, differentiate and even integrate maps. In addition, the spatial coincidence and juxtaposition of values among and within map layers create new mathematical operations, such as effective distance, optimal path routing, visual exposure density and landscape diversity, shape and pattern.

This PowerPoint with notes and online links to further reading is posted at

www.innovativegis.com/basis/Courses/SpatialSTEM/Workshop/

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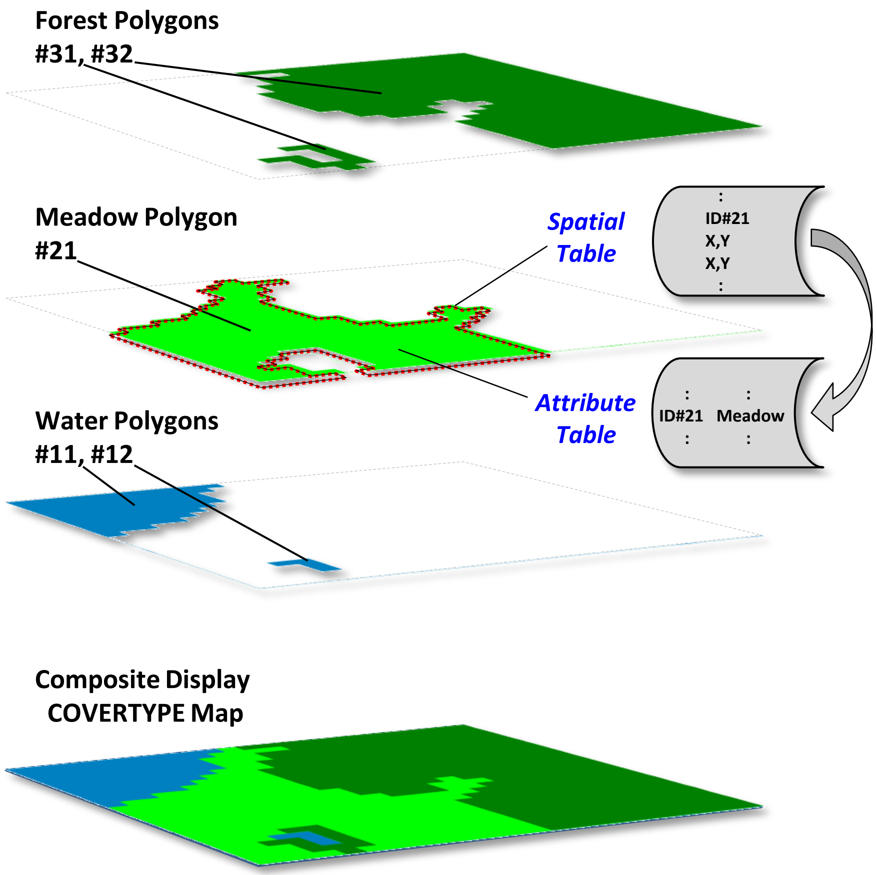
Vector vs. Raster Data Forms *(analytical perspective)*

Vector *(Spatial & Attribute Tables)*



Raster *(Grid Matrix)*

3 Forest
2 Meadow
1 Open Water

1	1	1	1	1	1	1	1	1	1	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3
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Mapping/Geo-Query *(Discrete, Spatial Objects)*

Where – a Spatial Table contains X,Y coordinates delineating the location of each point, line and polygon boundary

What – a linked Attribute Table contains text/values indicating the classification of each spatial object

(Continuous, Map Surfaces) Map Analysis/Modeling

Where – Cell Position in the matrix determines location within a continuous, regular grid

What – Cell Value in the matrix indicates the classification at that location

*…grid-based mapped data is "**Preconditioned for Analysis**" as spatial topology is inherent in the matrix*

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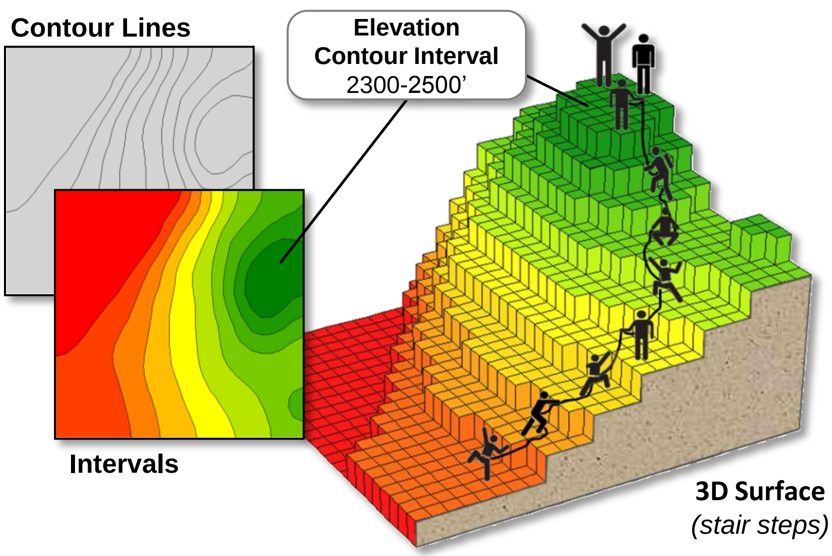
Grid-based Data Structure *(fundamental organizational concepts)*

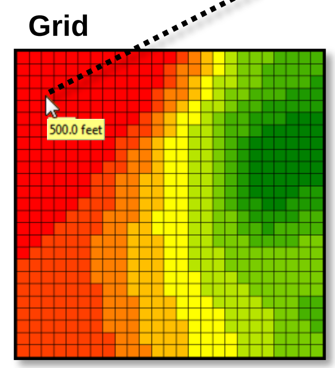
A ***Grid Map*** *consists of a matrix of numbers with a value indicating the characteristic/condition at each grid cell location—*

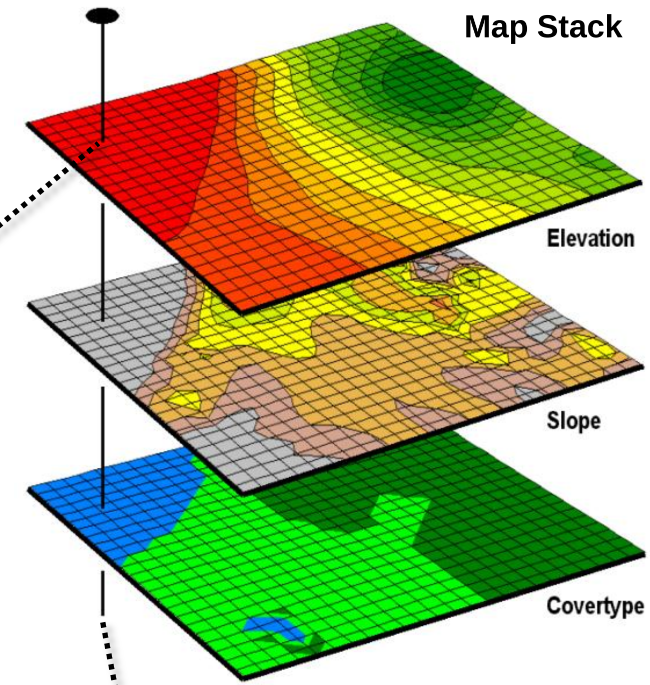
…forming a set of geo-registered ***Map Layers*** *or "****Map Stack****"*

Vector *(discrete)*



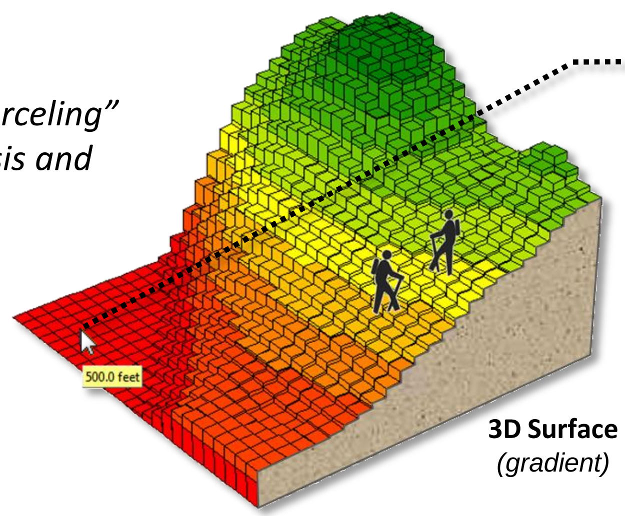
Raster *(continuous)*

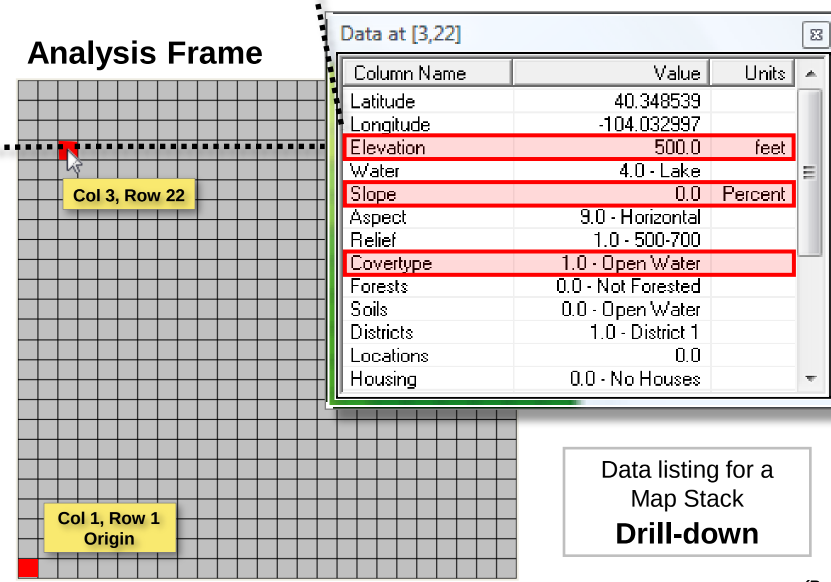




The ***Analysis Frame*** *provides consistent "parceling" needed for map analysis and extends* *__discrete__* *Points, Lines and Polygons…*

…to *__continuous__* ***Map Surfaces***





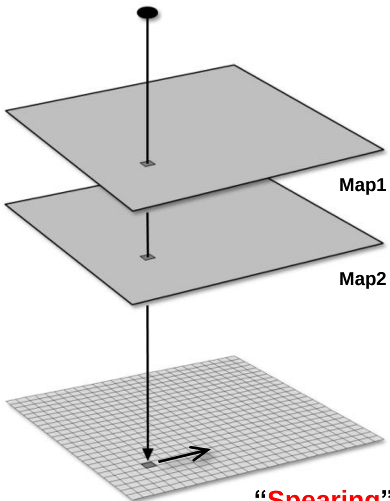
Column Name	Value	Units
Latitude	40.348539	
Longitude	-104.032997	
Elevation	500.0	feet
Water	4.0 - Lake	
Slope	0.0	Percent
Aspect	9.0 - Horizontal	
Relief	1.0 - 500-700	
Covertype	1.0 - Open Water	
Forests	0.0 - Not Forested	
Soils	0.0 - Open Water	
Districts	1.0 - District 1	
Locations	0.0	
Housing	0.0 - No Houses	

Data listing for a Map Stack
Drill-down

(Berry)

Grid-based Processing Structure *(data accessing and cyclic processing)*

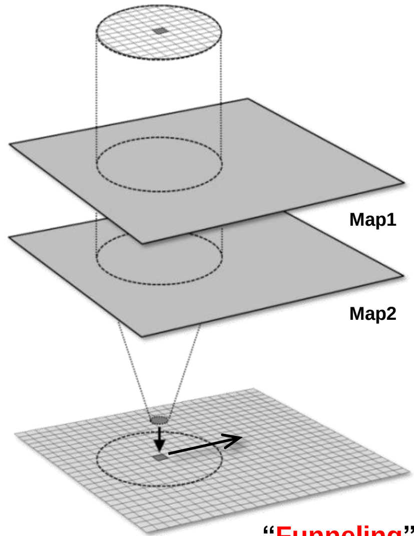
Local



"**Spearing**"

…collects data on a <u>cell-by-cell</u> basis and reports a single value on a <u>cell-by-cell</u> basis

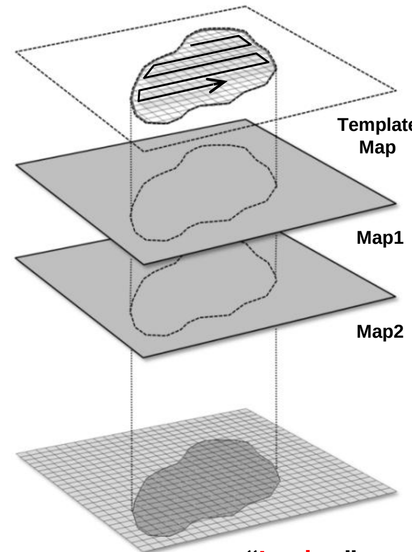
Focal



"**Funneling**"

…collects data on a <u>neighborhood</u> basis and reports a single value on a <u>cell-by-cell</u> basis

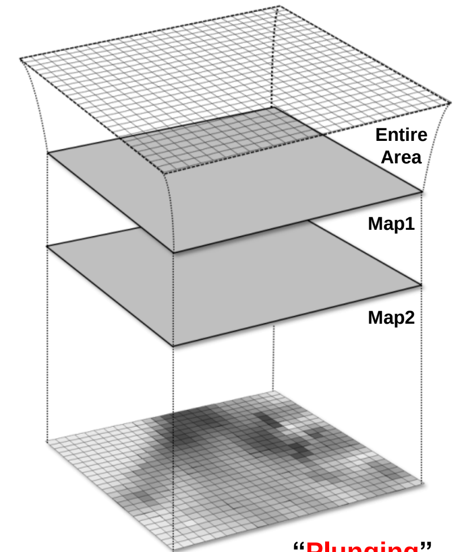
Zonal



"**Lacing**"

…collects data on a <u>region-wide</u> basis and reports summary on a <u>region-wide</u> basis

Global



"**Plunging**"

…collects data on a <u>map-wide</u> basis and reports results on a <u>map-wide</u> or <u>cell-by-cell</u> basis

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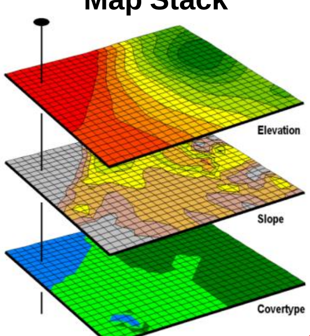
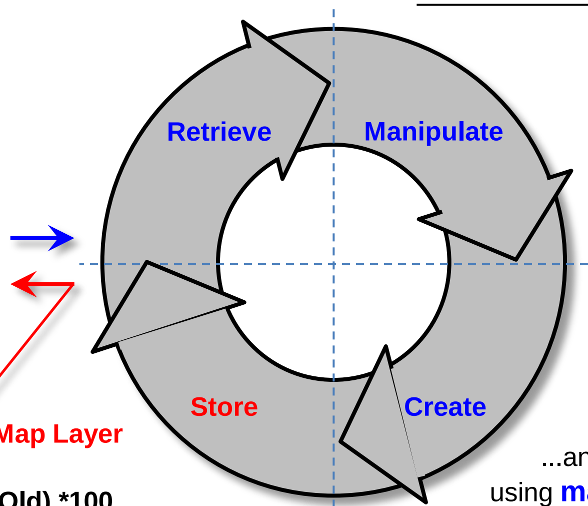
Map Stack





Intermediate Map Layer

%Change= ((Old – New) / Old) *100

Each **<u>processing step</u>** is accomplished by requiring—

1) **<u>Retrieval</u>** of one or more grid map layers from the map stack

2) **<u>Manipulation</u>** of that mapped data by an appropriate math/stat operation,

3) **<u>Creation</u>** of an <u>intermediate map layer</u> whose map values are derived as a result of that manipulation, and

4) **<u>Storage</u>** of that new map layer back into the map stack for subsequent processing.

...analogous to evaluating "**nested parentheticals**" in traditional algebra, except using **map variables** composed of thousands of spatially organized numbers

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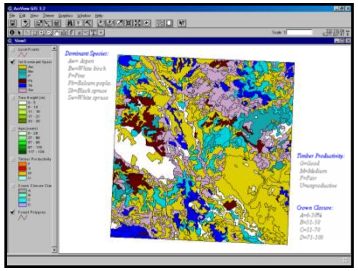
Overview of Map Analysis Approaches

(Spatial Analysis and Spatial Statistics)

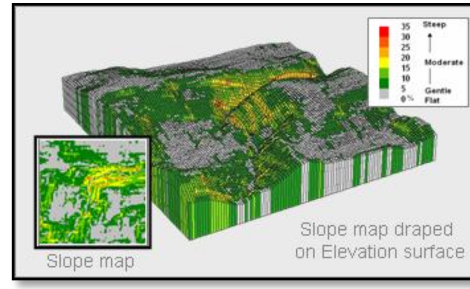
Traditional GIS



Forest Inventory Map

- **Points, Lines, Polygons**
- **Discrete Objects**
- **Mapping and Geo-query**

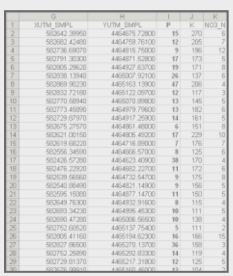
Spatial Analysis

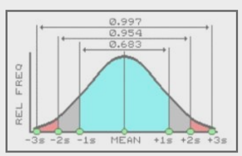


Elevation (Surface)

- **Cells, Surfaces**
- **Continuous Geographic Space**
- **Contextual Spatial Relationships**

Traditional Statistics

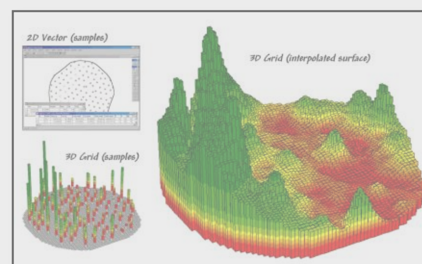




Minimum= 5.4 ppm
Maximum= 103.0 ppm
Mean= 22.4 ppm
StDEV= 15.5

- **Mean, StDev (Normal Curve)**
- **Central Tendency**
- **Typical Response (scalar)**

Spatial Statistics



Spatial Distribution (Surface)

- **Map of Variance (gradient)**
- **Spatial Distribution**
- **Numerical Spatial Relationships**

…next session

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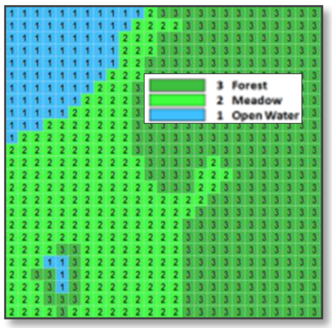
Spatial Analysis Operations *(Geographic Context)*

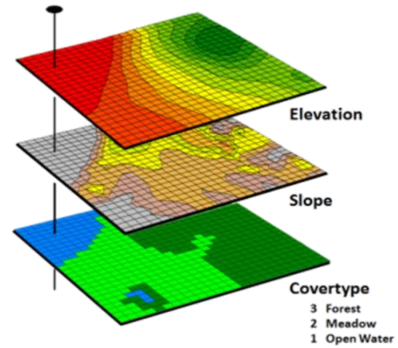
GIS as "Technical Tool" *(Where is What)* vs. "**Analytical Tool**" *(Why, So What and What if)*



Grid Layer

Spatial Analysis

Map Stack



Spatial Analysis extends the basic set of discrete map features (points, lines and polygons) to map **surfaces** that represent continuous geographic space as a set of contiguous grid cells (matrix), thereby providing a **Mathematical Framework** for *map analysis* and *modeling* of the

Contextual Spatial Relationships within and among grid map layers

Map Analysis Toolbox

Mathematical Perspective:

…let's consider some examples →

Basic GridMath & Map Algebra *(+ - * /)*

Advanced GridMath *(Math, Trig, Logical Functions)*

Map Calculus *(Spatial Derivative, Spatial Integral)*

Map Geometry *(Euclidian Proximity, Effective Proximity, Narrowness)*

Plane Geometry Connectivity *(Optimal Path, Optimal Path Density)*

Solid Geometry Connectivity *(Viewshed, Visual Exposure)*

✓ ***Unique Map Analytics*** *(Contiguity, Size/Shape/Integrity, Masking, Profile)*

✓ ***Unique spatial operations***

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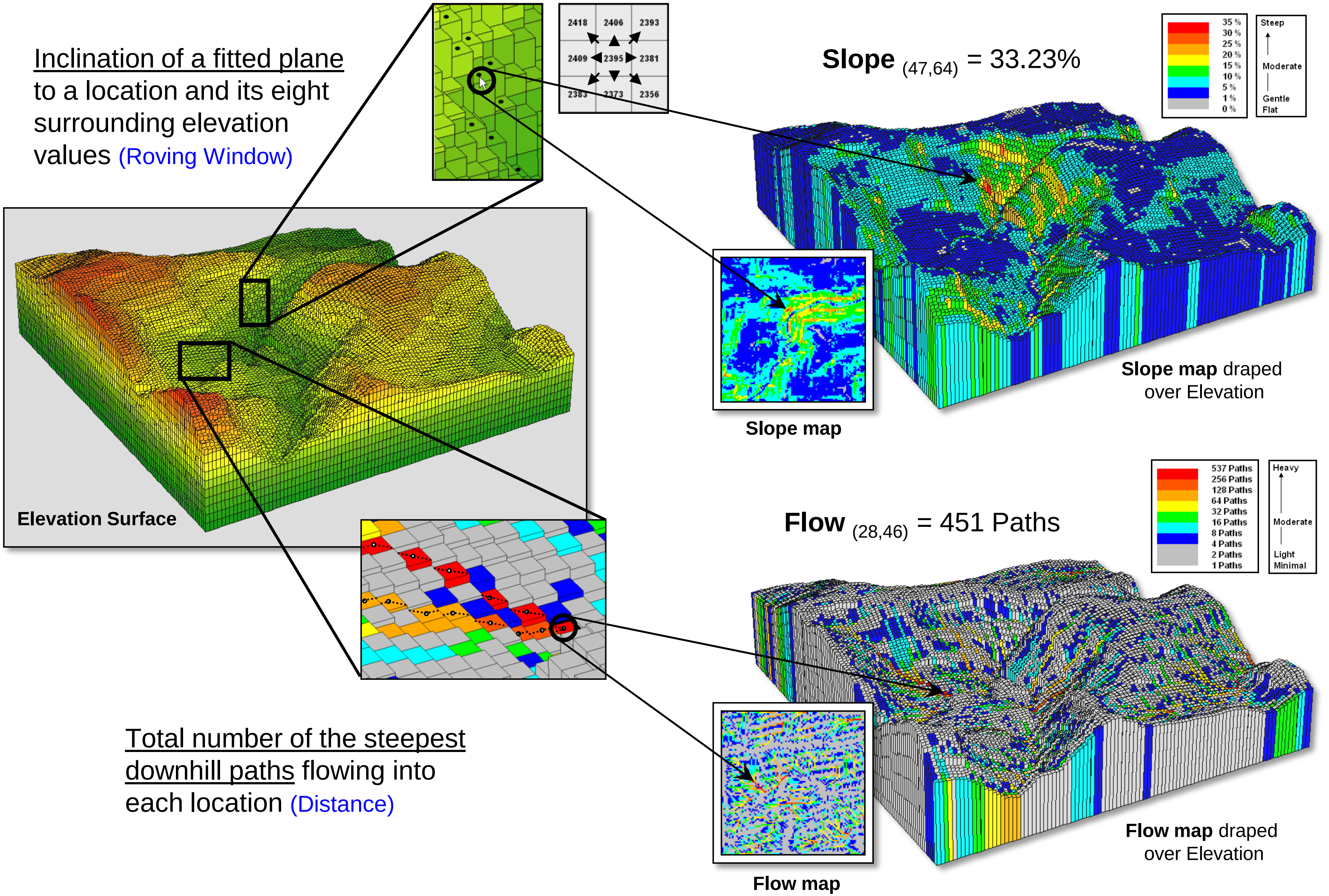
Calculating Slope and Flow *(Spatial Derivative; Optimal Path Density)*

Inclination of a fitted plane to a location and its eight surrounding elevation values (Roving Window)

Total number of the steepest downhill paths flowing into each location (Distance)



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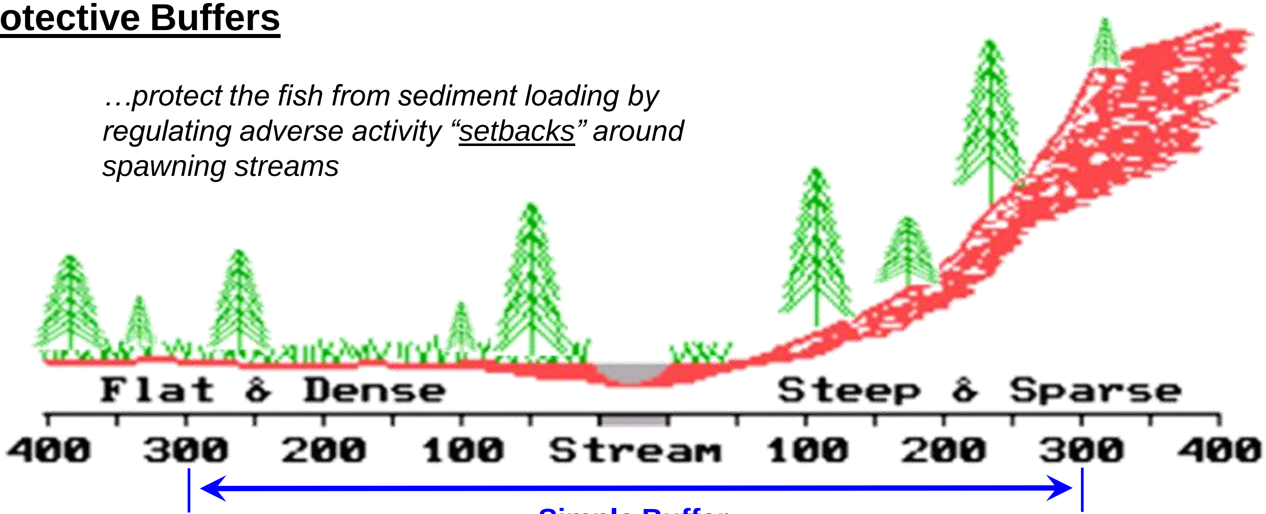
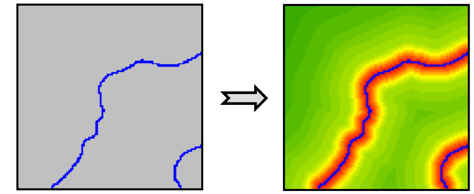
Deriving Erosion Potential *(terrain slope and surface flow)*

Protective Buffers

…protect the fish from sediment loading by regulating adverse activity "setbacks" around spawning streams



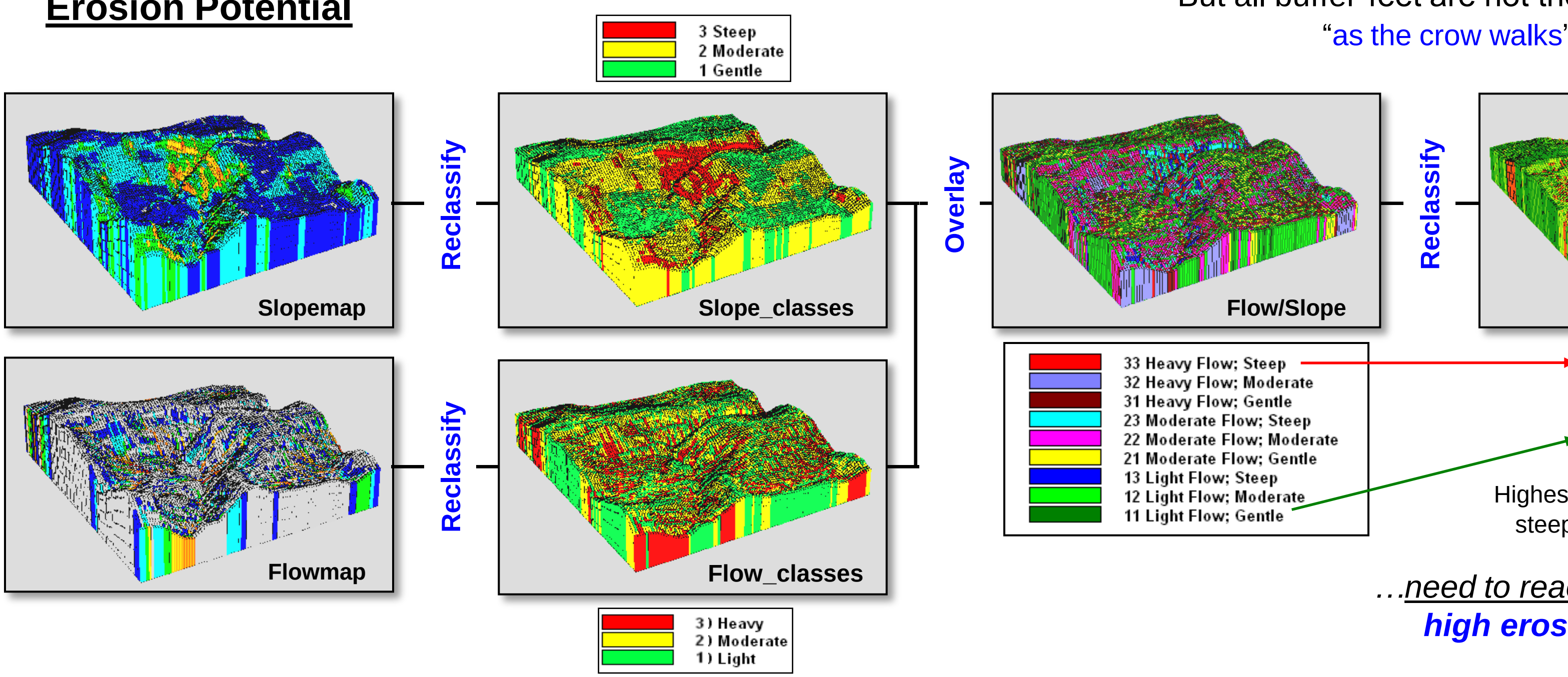


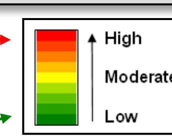
Streams | Simple Buffer

…simple distance buffers
"as the crow flies"

Erosion Potential

But all buffer-feet are not the same…
"as the crow walks"





Highest **erosion potential** on steep slopes with heavy flows

…need to reach farther in areas of ***high erosion potential*** →

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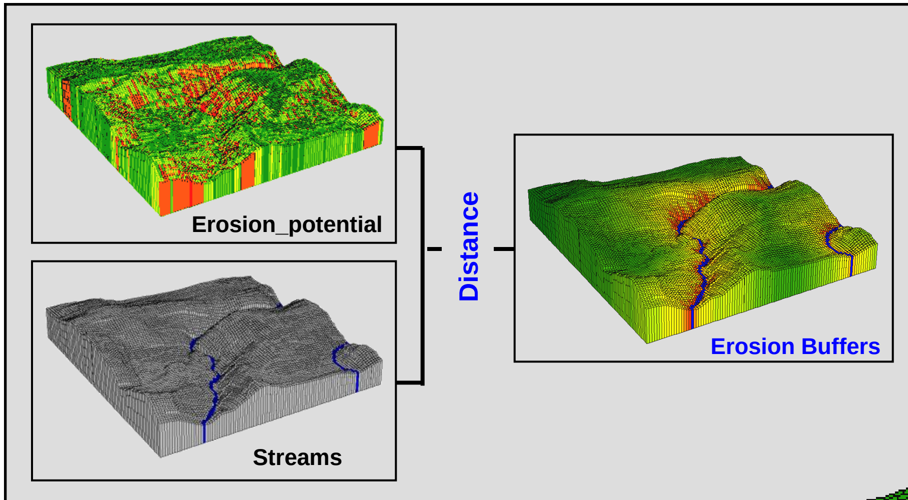
Calculating Effective Distance (variable-width buffers)



Distance away from the streams is a function of the erosion potential (Flow/Slope Class)

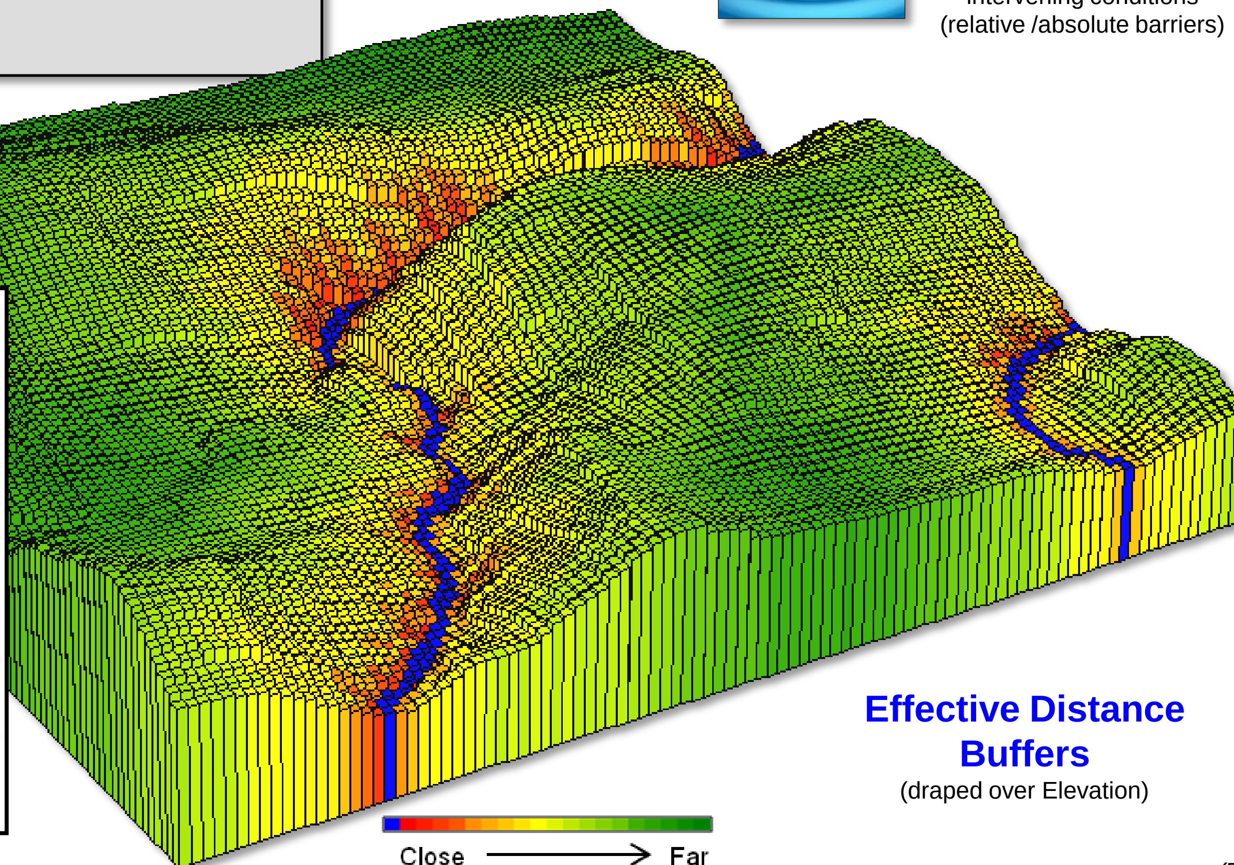
…with intervening heavy flow and steep slopes computed as effectively closer than simple distance

"as the crow walks"

…respecting relative and absolute barriers to movement

Splash Algorithm

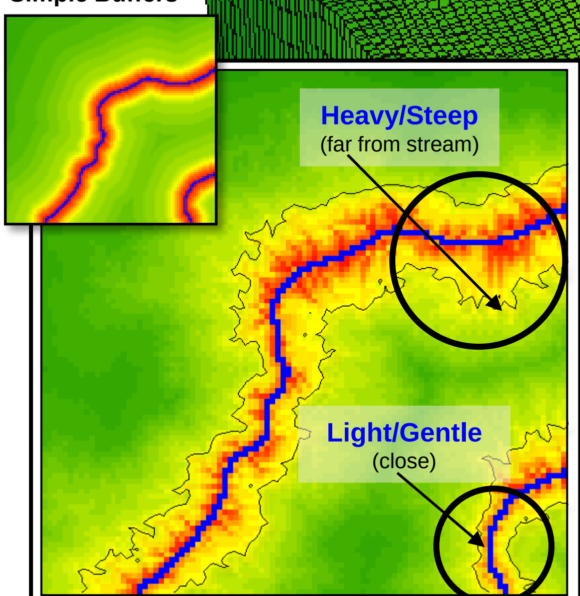
…propagating distance waves respond to intervening conditions (relative /absolute barriers)



Effective Distance Buffers

(draped over Elevation)

Simple Buffers



(digital slide show VBUFF) **Effective Buffers**

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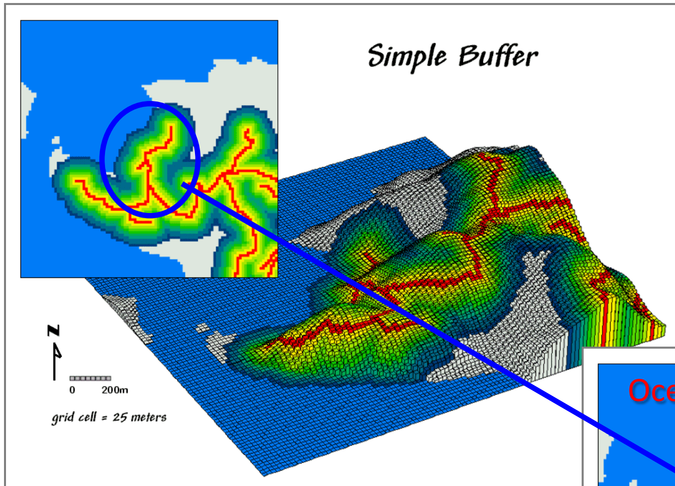
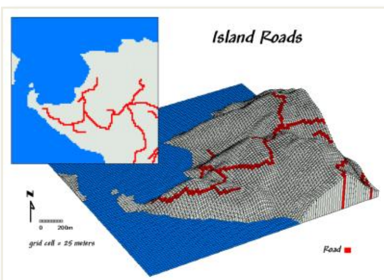
Variable-Width Buffers *(Simple vs. Effective "clipped and uphill")*

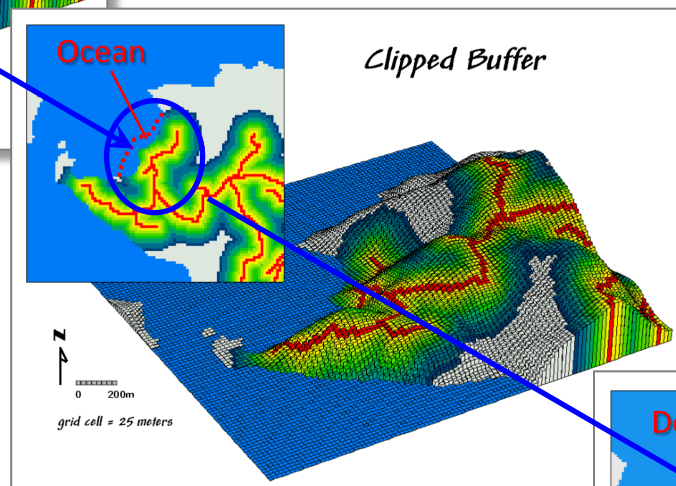
Spatial Analysis:

Basic GridMath & Map Algebra
Advanced GridMath
Map Calculus
Map Geometry
Plane Geometry Connectivity
Solid Geometry Connectivity
Unique Map Analytics

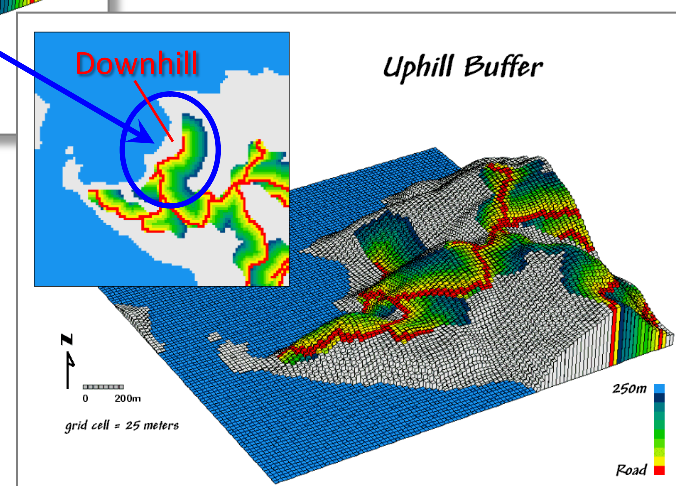


Simple Buffer – "as-the-crow-flies" proximity to the road; no absolute or relative barriers are considered; dark blue line indicates the full simple buffer reach (polygon)





Clipped Buffer – simple proximity for just land areas



Uphill Buffer – simple proximity to the road for just the areas that are uphill from the road; absolute barrier (uphill only– absolutely no downhill steps)

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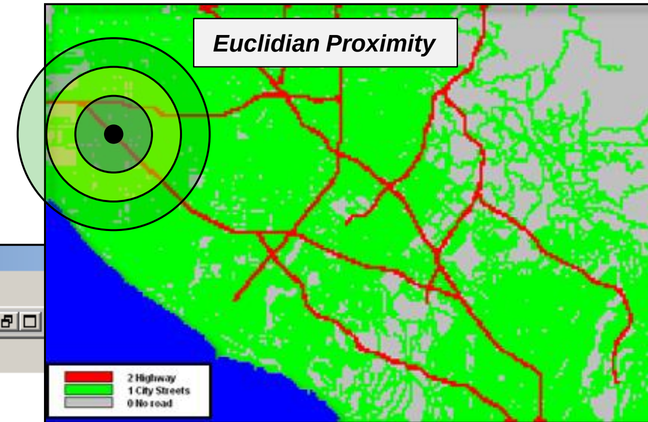
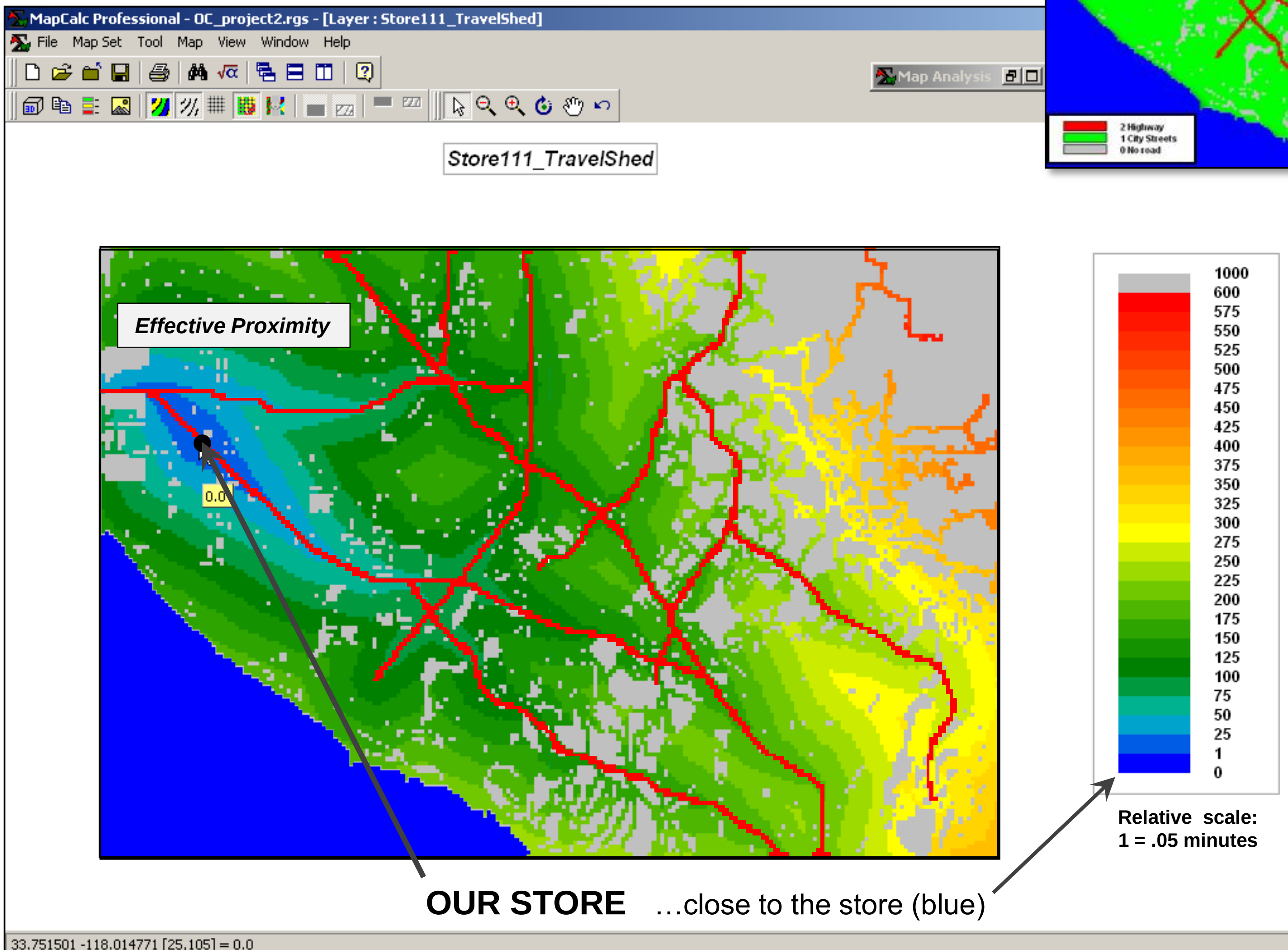
Calculating Travel-time *(Euclidian Proximity, Effective Proximity)*

A store's **Simple Proximity** identifies "rings" of increasing <u>geometric distance</u> — "concentric circles"



A store's **Travelshed** identifies the relative <u>driving time</u> from every location to the store — analogous to a "watershed"



Splash Algorithm

…propagating distance waves respond to intervening conditions (relative/absolute barriers)

Spatial Analysis:

Basic GridMath & Map Algebra
Advanced GridMath
Map Calculus
Map Geometry
Plane Geometry Connectivity
Solid Geometry Connectivity
Unique Map Analytics

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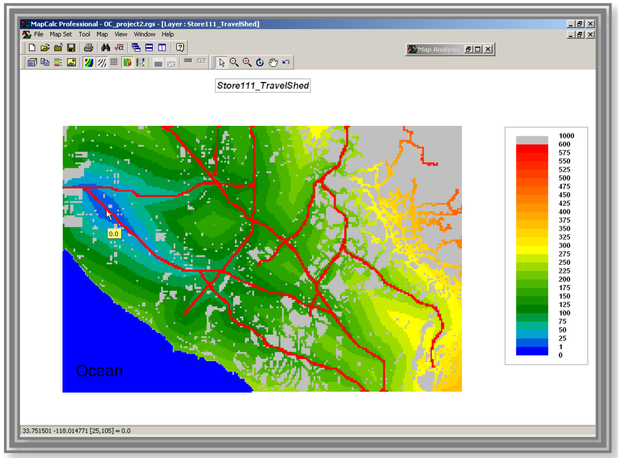
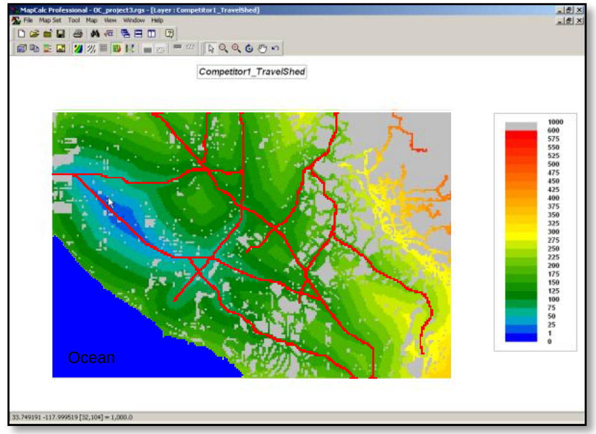
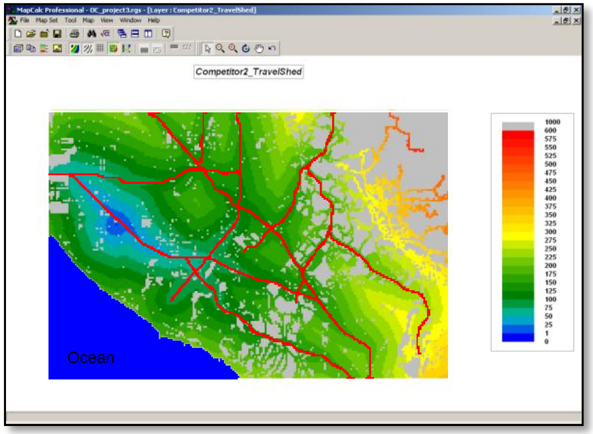
Travel-Time for Competitor Stores *(Euclidian Proximity, Effective Proximity)*



Our Store (#111)

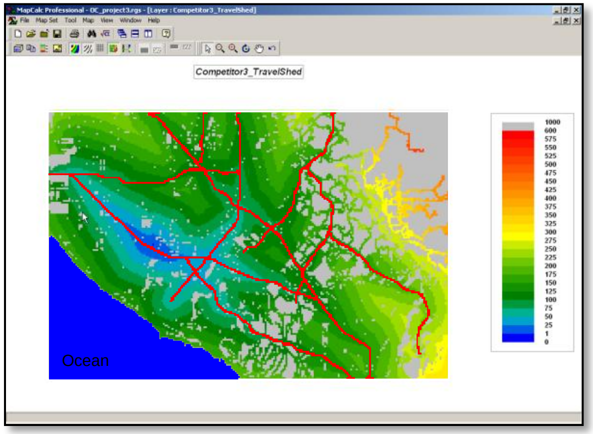


Competitor 1

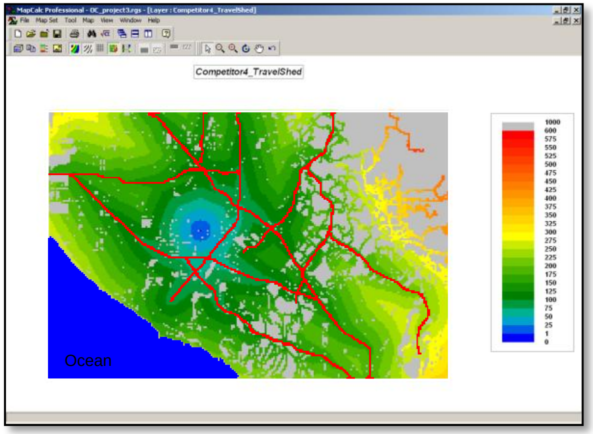


Competitor 2

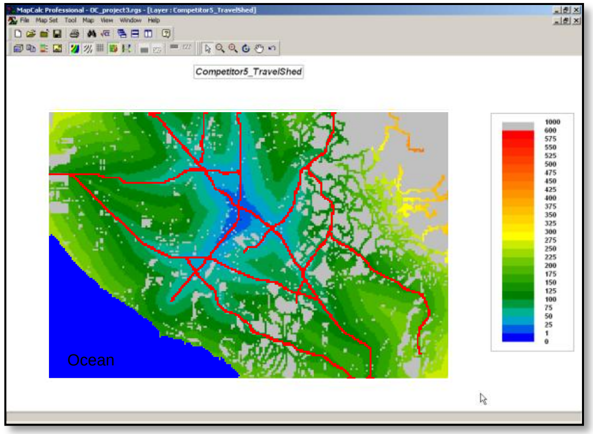
Competitor 3



Competitor 4



Competitor 5



Travel-Time surfaces from several stores
treating highway travel as four times faster than city streets

Blue tones indicate locations that are close to a store (estimated twelve minute drive or less).
Customer data can be appended with travel-time distances and analyzed for spatial relationships in sales and demographic factors.

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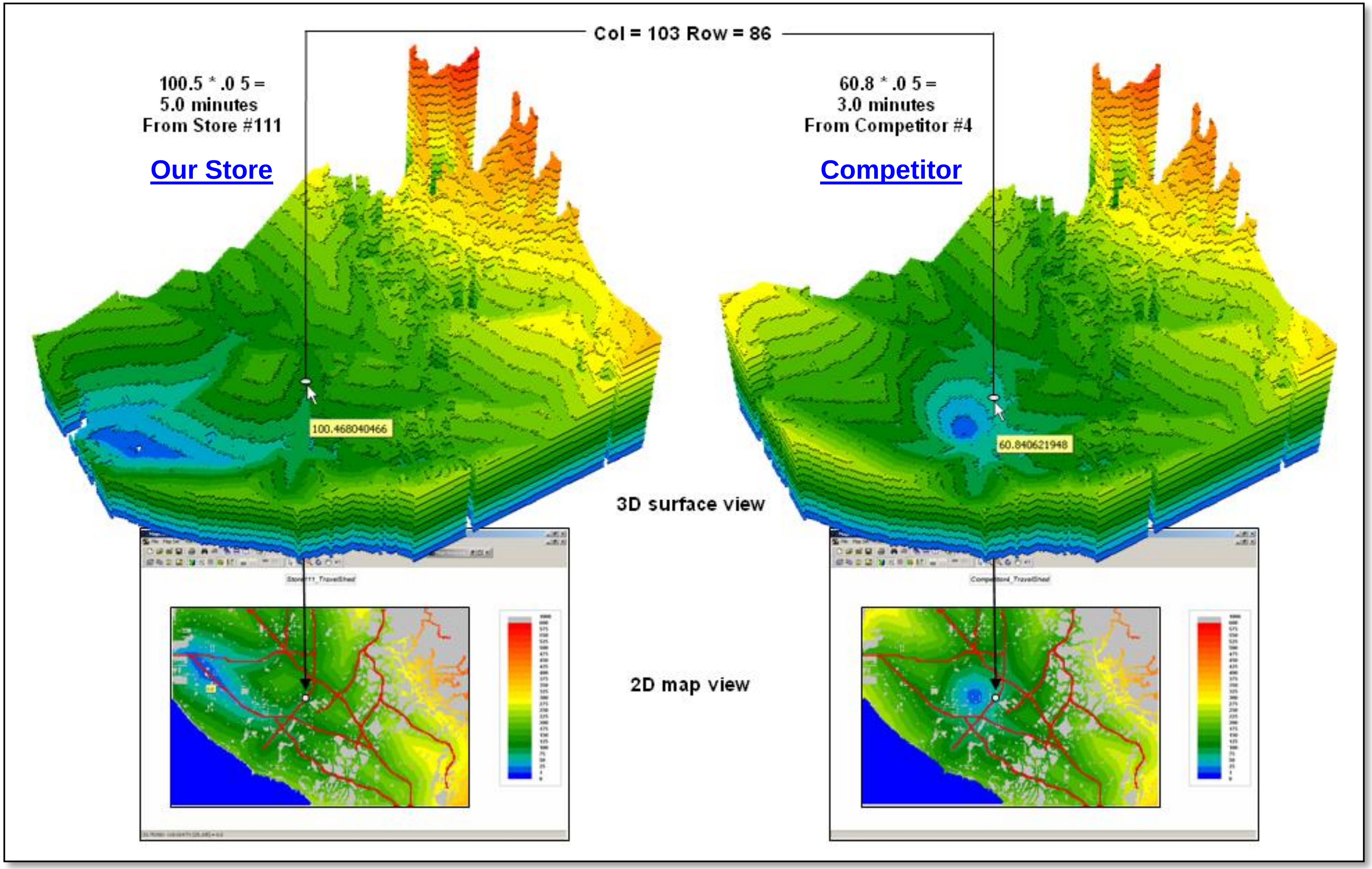
Travel-Time Surfaces (Our Store & Competitor #4)

Blue tones indicate locations that are close to a store (estimated twelve minute drive or less). Increasingly warmer tones form an ever increasing **bowl-like gradient** (accumulation surface) with larger travel-time values identifying locations that are farther away.



(Berry)

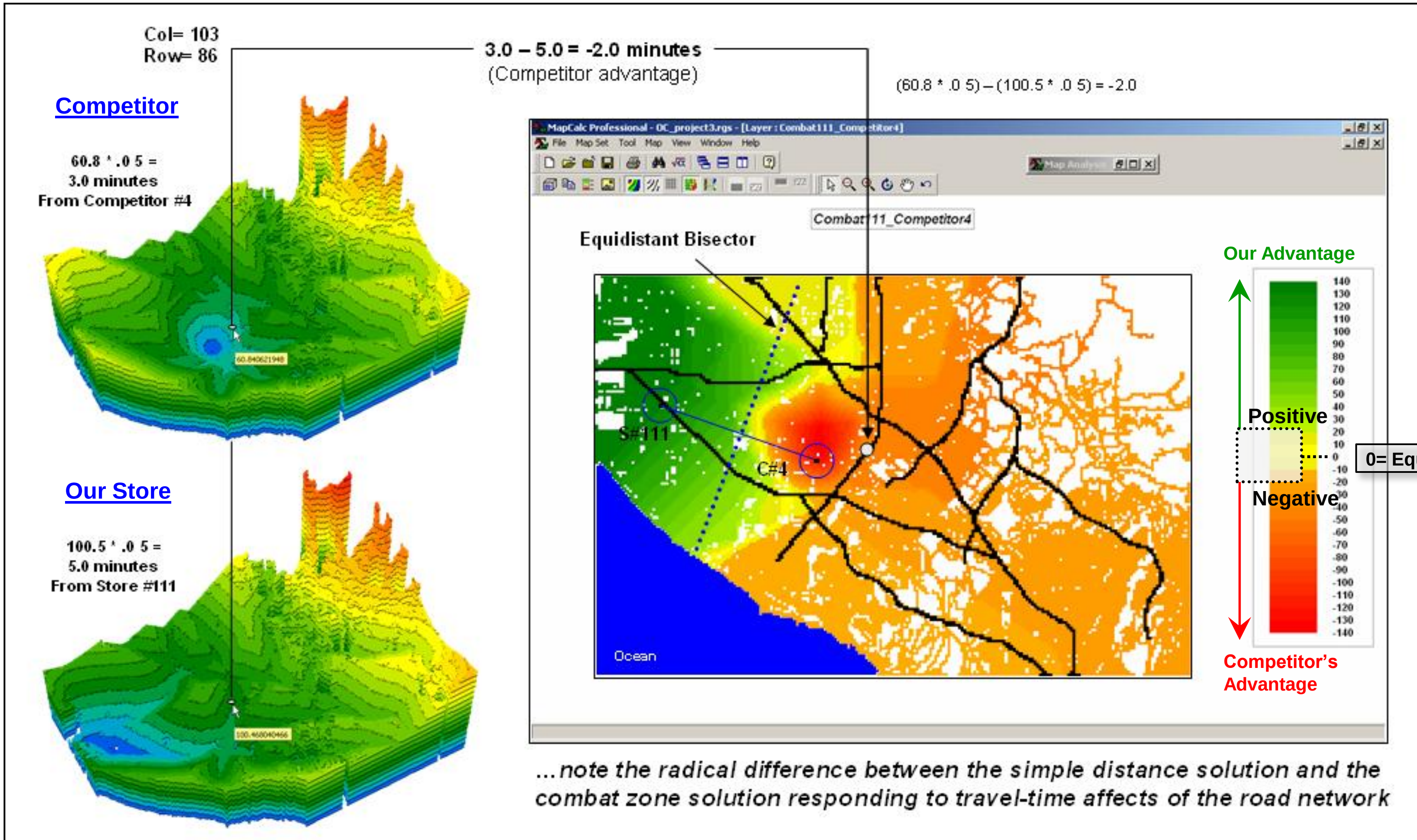
Competition Map (Combat Zone between Our Store & Competitor #4)

The travel-time surfaces for two stores can be compared (subtracted) to Identify the **relative access advantages** throughout the project area.

Zero values indicate the same travel-time to both stores (equidistant travel-time) …yellow identifies the Combat Zone ; green Our Store advantage; red Competitor #4 advantage

Spatial Analysis:

Basic GridMath & Map Algebra
Advanced GridMath
Map Calculus
Map Geometry
Plane Geometry Connectivity
Solid Geometry Connectivity
Unique Map Analytics



…note the radical difference between the simple distance solution and the combat zone solution responding to travel-time affects of the road network

(See "***Location, Location, Location: Retail Sales Competition Analysis***," posted at **www.innovativegis.com/basis/present/GW06_retail/GW06_Retail.htm**)

(Berry)

Map Geometry & Connectivity Techniques *(Travel-time and Optimal Path)*

<u>Step 1</u>) Establish **off-road hiking friction** that considers the relative ease of hiking through various cover types (in minutes/cell) as 2= meadow, 5= forest relative barriers and 0= open water (absolute barrier)

<u>Step 2</u>) Establish **on-road bicycling** friction as 1= easiest with 0= all non-road areas

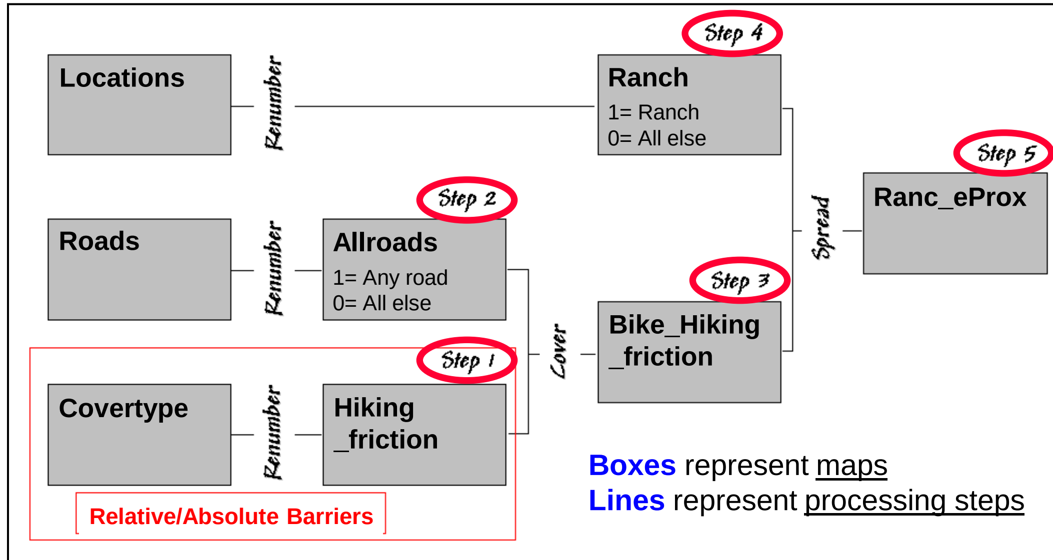
<u>Step 3</u>) **Combine** the on- and off-road friction maps such that the on-road friction takes precedent

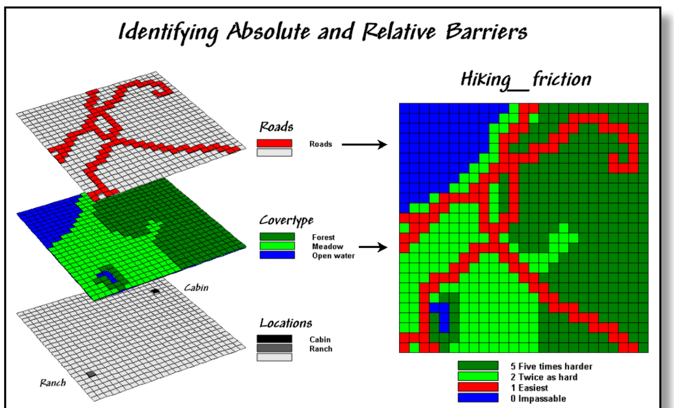
<**Friction Map**>

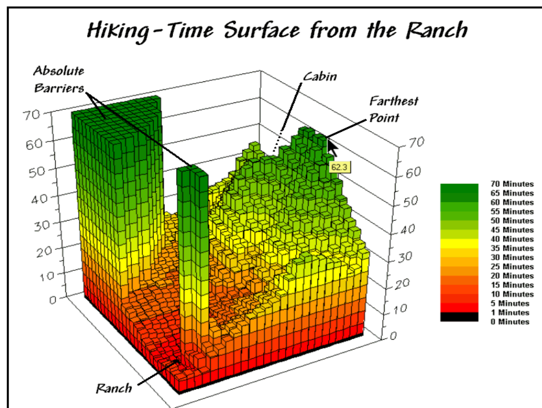
<u>Step 4</u>) **Isolate** the starting location (ranch)

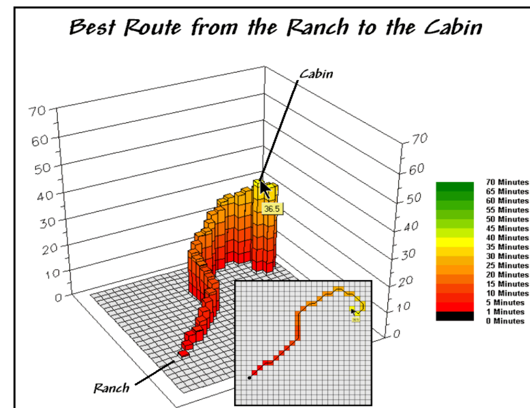
<u>Step 5</u>) **Derive** the effective proximity (Travel-time) from the Ranch to everywhere in the project area

<**Accumulation Surface**>









…steps 1-3) **Friction Map**– identifies <u>the relative ease of travel</u> through each map location (grid cell).

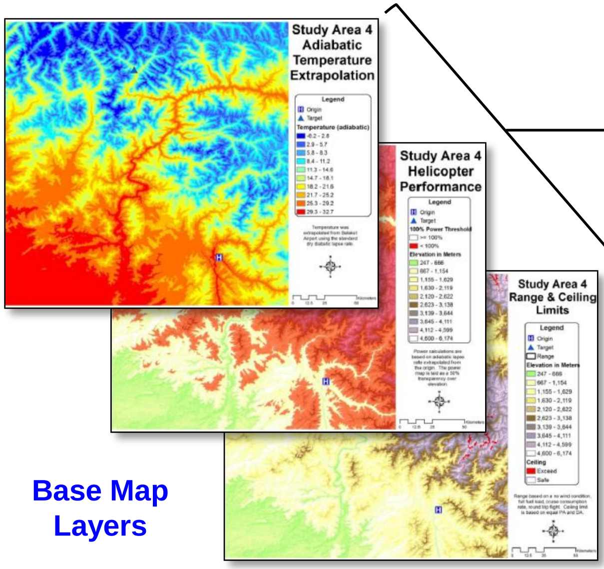
…steps 4-5) **Accumulation Surface**– identifies the <u>Travel-time</u> (eProximity) from the Ranch to everywhere.

Accumulation Surface– identifies the <u>Optimal Path</u> (shortest/quickest route) from a location back to the ranch. The "*steepest downhill path*" retraces the effective distance wavefront that got there first.

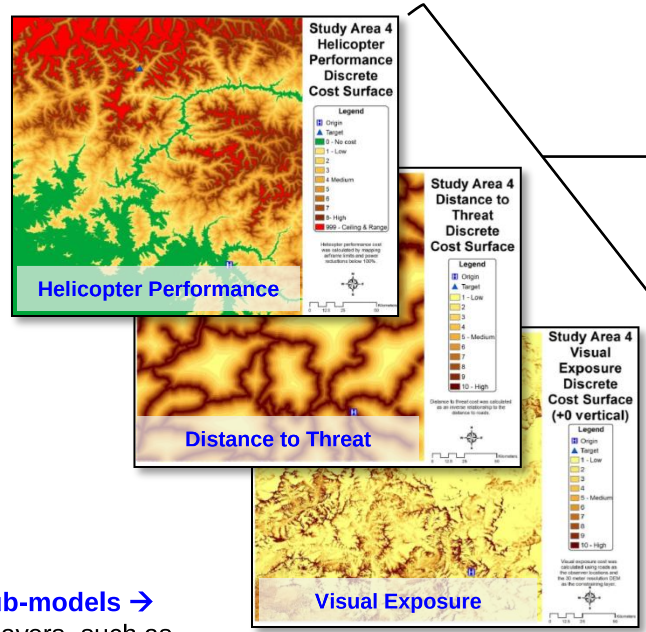
Optimal Corridor Analysis– adding two effective proximity surfaces (start and end) identifies the optimal path as the minimum value and all other values as the added "opportunity cost" of forcing a route through any location in the project area.

(Berry)

Optimal Path/Corridor Analysis *(minimize detection/risk model)*

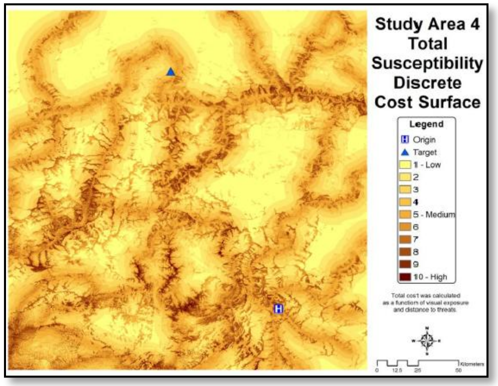


Base Map Layers



Discrete Cost Surface sub-models →
Utilize additional base map layers, such as Night Lights, Population, Roads, Topography, Vegetation, etc. are used for the other

LOW Prefer Avoid **HIGH**



Overall Discrete Cost Surface
(Susceptibility of Detection)

Least Cost Path

Optimal Path and Corridor
(Minimizes Susceptibility of Detection)

Spatial Analysis:

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Advanced GridMath
Map Calculus
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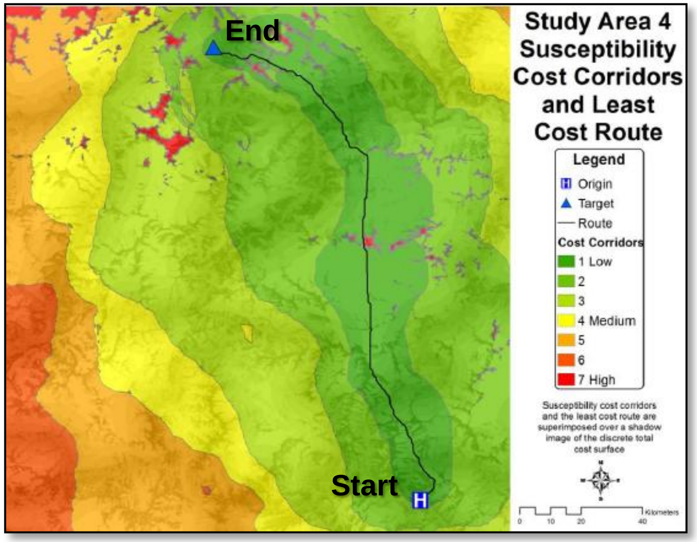
Solid Geometry Connectivity
Unique Map Analytics



"…GIS optimized flight routing plans that
minimize helicopter susceptibility to detection
(maximize capability to avoid threats)…"

…figures from "Susceptibility Modeling and Mission Flight Route Optimization in a Low Threat, Combat Environment"
by USAF Lt. Colonel Brett J. Machovina, June 2010, doctoral dissertation, University of Denver

(Berry)

Map Geometry *(Simple Euclidian and Effective Proximity)*

<u>Basic</u> Operations (Static)

✓ **Simple Proximity** as the "crow flies" counting <u>cell lengths</u> as it moves out as a wave front

✓ **Effective Proximity** as the "crow walks" in not necessarily in straight lines that respect <u>absolute/ relative</u> impedance to movement

Operation Specifications

✓ **Null** identifies a "Thru" surface value identifying ignored locations

✓ **To** indicates maximum distance of movement

✓ **Thru** map identifying the relative/absolute impedance (discrete cost)

✓ **Over** respects movement <u>Uphill, Downhill or Across</u> a specified surface map considering a "guiding surface" such as Elevation

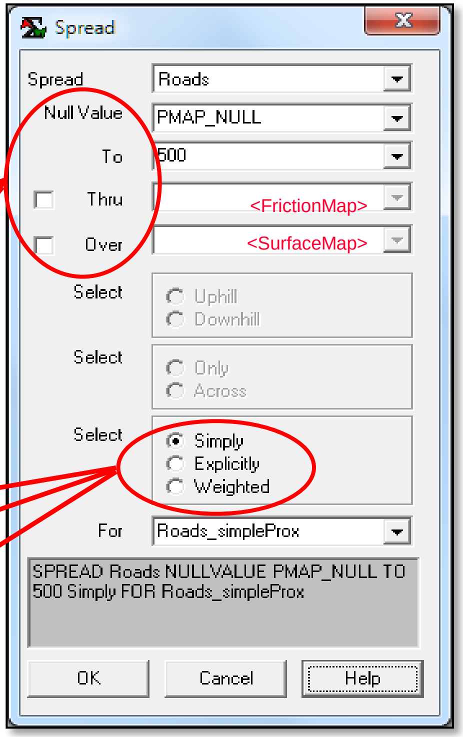
✓ **Simply** starts "counting" simple/effective movement from 1

✓ **Explicitly** starts "counting" from the grid location's value thereby creating a "stepped accumulation surface"

✓ **Weighted** the starter cell's value is used as an additional weight to generate a "gravity model" solution

✓ **Back Link** stores a starter ID# identifying the <u>closest starter location</u>)

*<u>Spread</u> operation in **MapCalc Learner***



<u>EucDistance</u> operation
<u>CostDistance</u> operation
*in **Spatial Analyst** have similar capabilities*

<u>Advanced</u> Operations (Dynamic)

✓ **Accumulation** (<u>Total accumulated</u> movement in #cells)

✓ **Momentum** (<u>Net accumulated</u> movement considering increases/decreases in speed)

✓ **Direction** (<u>Look-up table</u> determining the effective impedance as a function movement direction, such as uphill or downhill slopes)

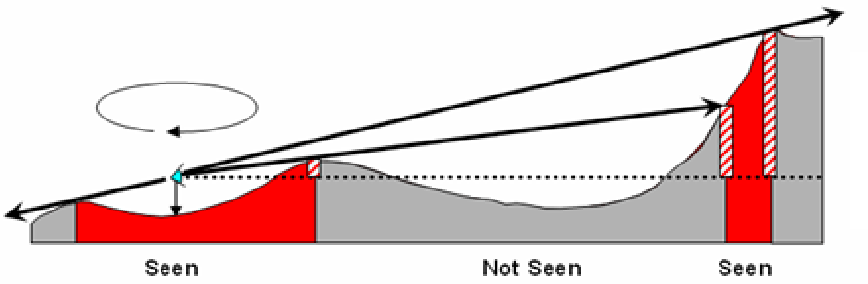
www.innovativegis.com/basis/MapAnalysis/Topic25/Topic25.htm

Beyond Mapping III

***Topic 25:** Calculating Effective Distance and Connectivity*

(Berry)

Calculating Visual Connectivity *(sequentially assessing the tangent)*



***Visual Connectivity* is like a rotating search light at a "viewer" location…**

Splash Algorithm

…propagating tangent waves carrying the "rise to run" ratio (tangent)

Spatial Analysis:

Basic GridMath & Map Algebra
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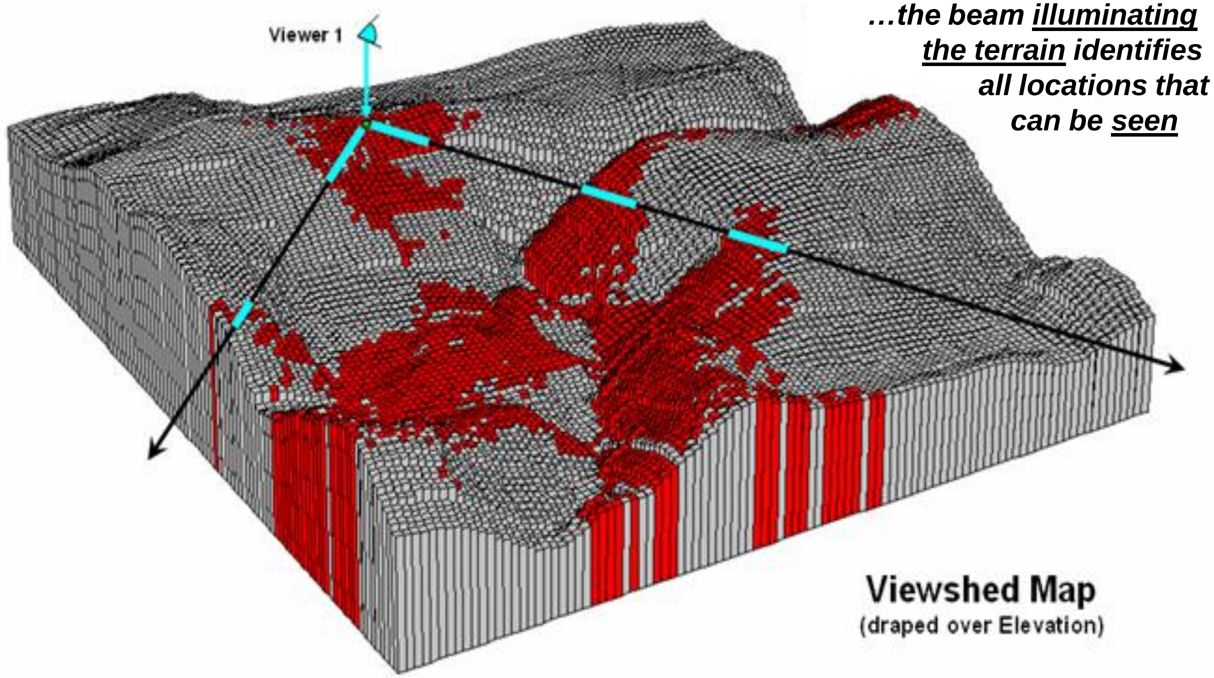
Plane Geometry Connectivity
Solid Geometry Connectivity
Unique Map Analytics

…like proximity, **Visual Connectivity** starts somewhere (starter cell) and moves through geographic space by steps (wave front) noting if the ratio of rise in elevation versus the distance away (**tangent**) is greater than any of the previous ratios, the location is marked as seen



…the beam illuminating the terrain identifies all locations that can be seen

Viewshed

Binary Map at least one viewer location "sees" a map location (**binary** seen or not seen)

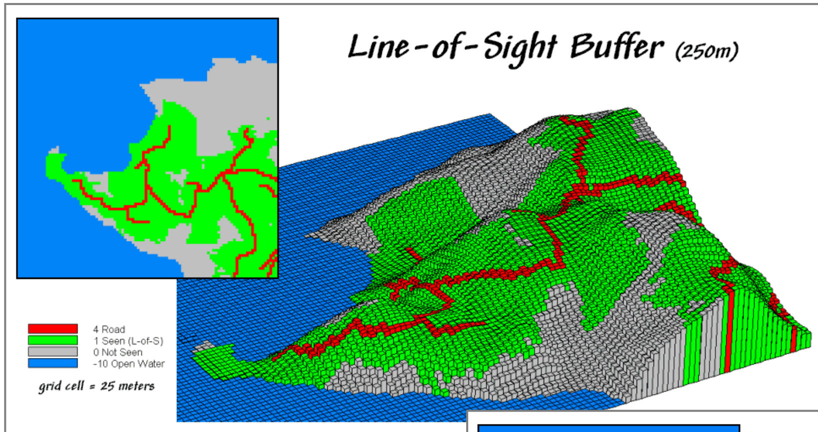
Visual Exposure

Density Surface – counts the number of "viewers" that see each map location (**relative density**)

Weighted Density surface – adds the viewer cell value (**relative importance**)

(Berry)

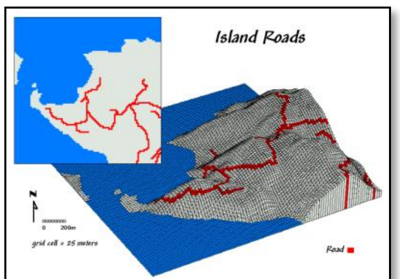
Variable-Width Buffers *(line-of-sight connectivity)*

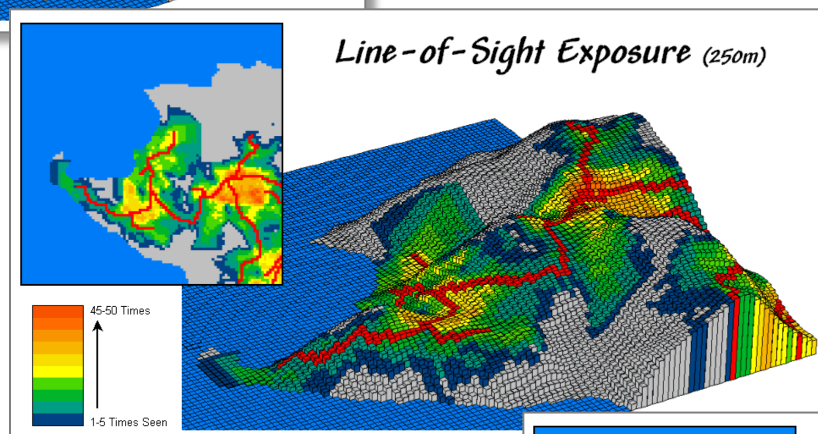


Line-of-Sight Buffer– identifies land locations within 250m that can be <u>seen from the road</u>…

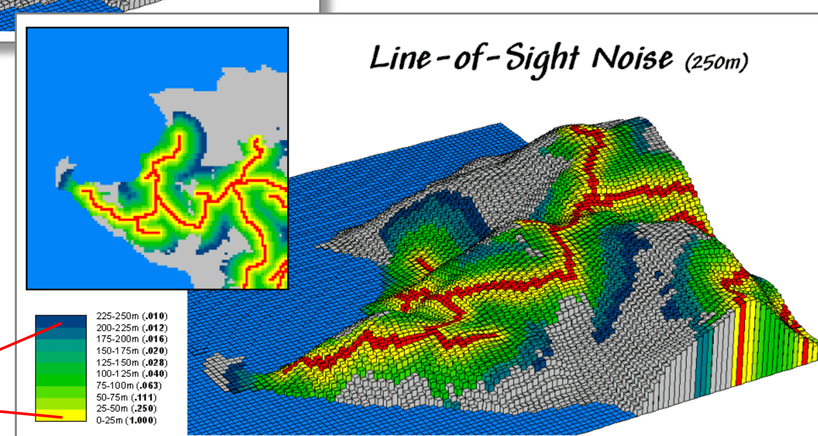
250m "viewshed" of the road

(clipped to land area)





Line-of-Sight Exposure– notes the <u>number of times</u> each location in the buffer is seen



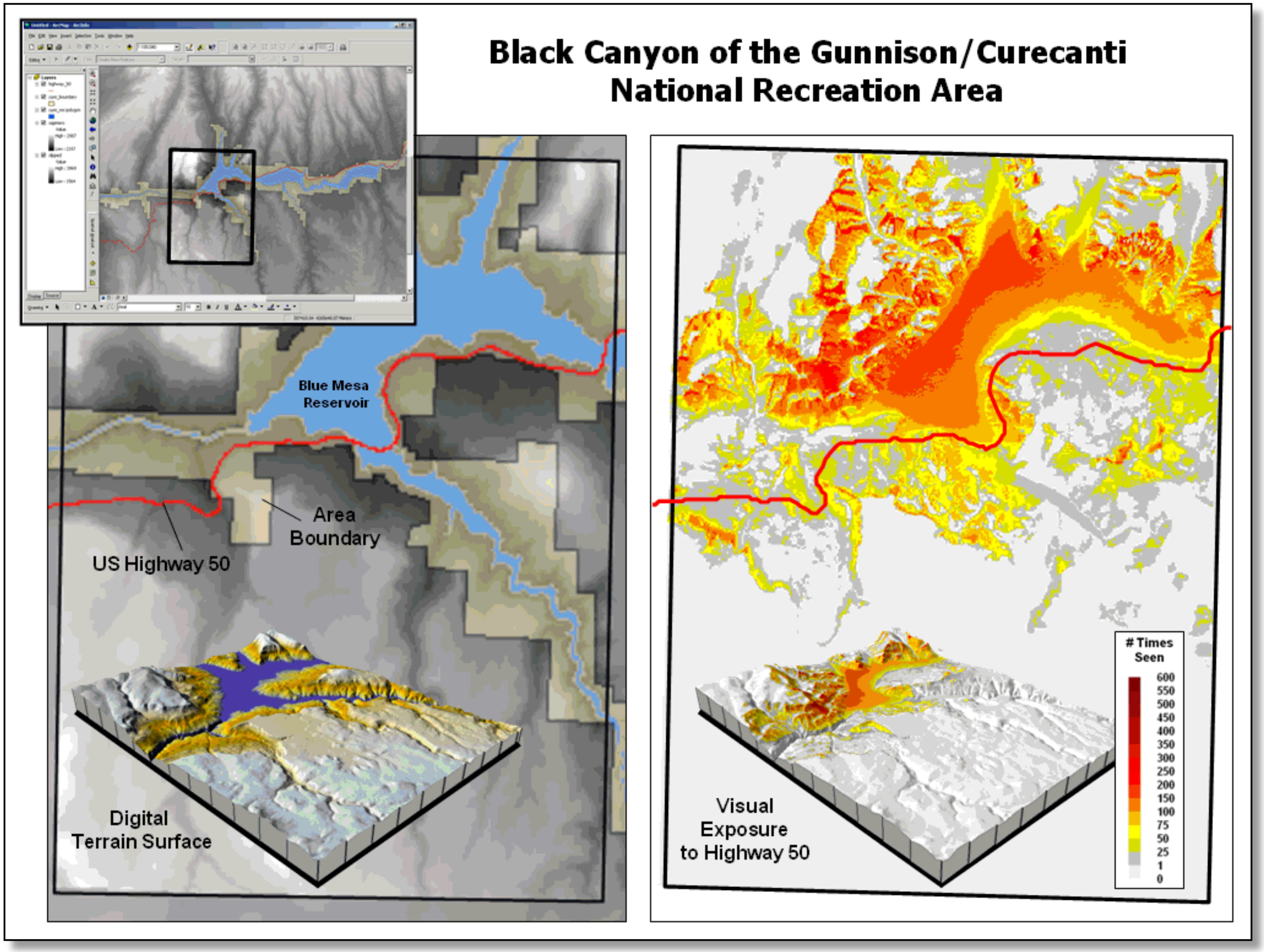
Line-of-Sight Noise– locations hidden behind a ridge or farther away from a source (road) "hear" greatly reduced <u>noise levels</u>

…$1/D^2$ decay function

Compute **Exposure_Map** * **Noise_map** for a relative **Noise Irritability Index**

(Berry)

Visual Exposure Analysis *(visual vulnerability and aesthetic maps)*



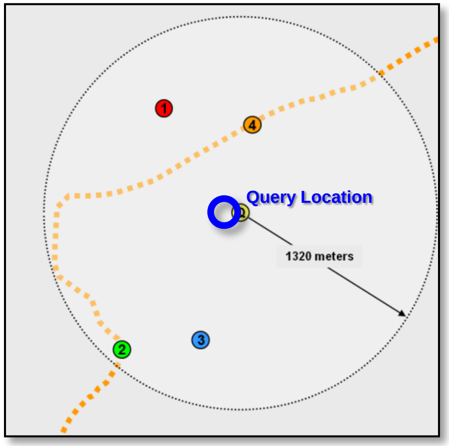
Weighted Visual Exposure map for an ongoing visual assessment in a national recreation area. The project developed visual vulnerability maps from the reservoir and a major highway (viewer locations) running through the area.

In addition, an **Aesthetic Map** was generated based on overall visual exposure to pretty and ugly places.

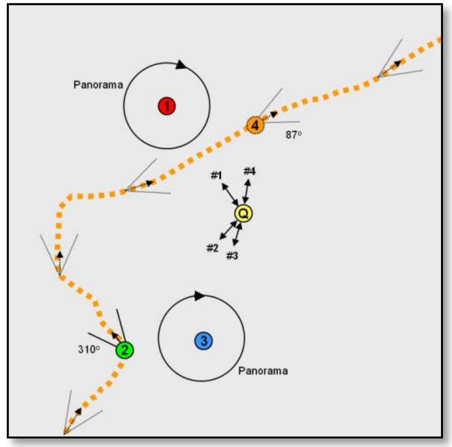
(Senior Honors Thesis by University of Denver Geography student Chris Martin, 2003)

(Berry)

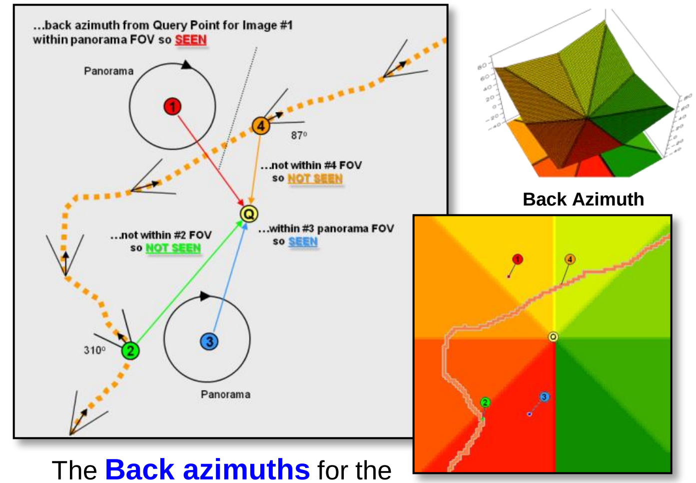
Spatially Accessing Relevant Images *(Back Azimuth and FOV)*



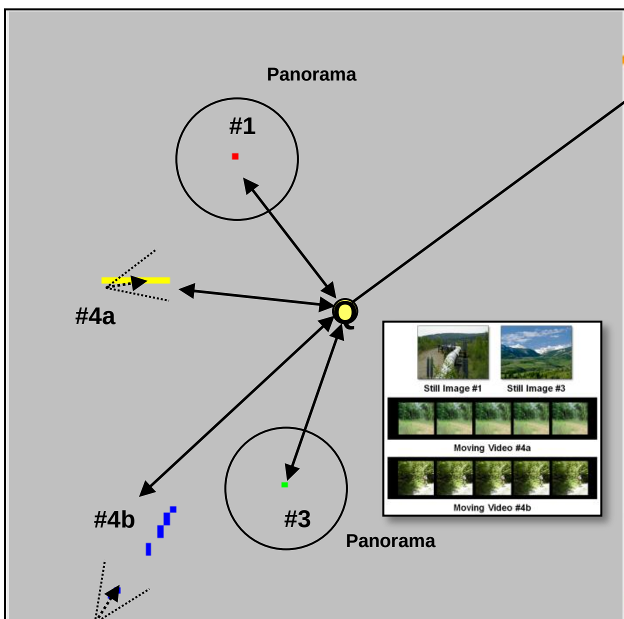
User identifies a location of interest…then specifies a **buffer distance** …and the images meeting the query are listed.



The direction to the imagery locations must be considered …with respect to the direction of the **camera's optical axis and field of view**.



The **Back azimuths** for the camera locations …are compared with the **optical axis/field of view** to determine if the camera is pointed toward the query point. Candidate imagery not oriented toward the query point are eliminated.

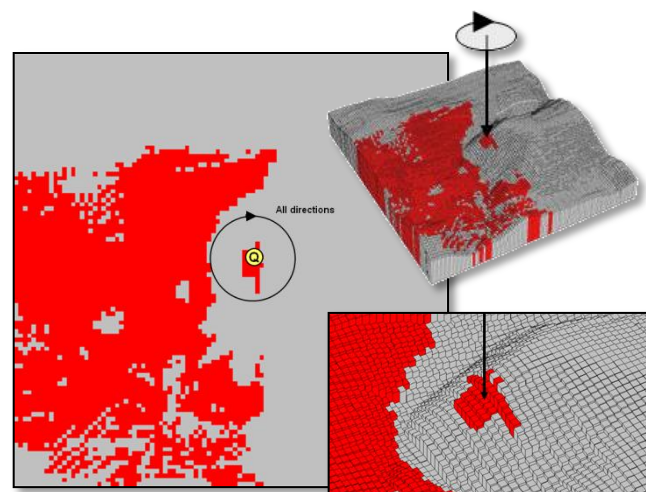


…***click***– and the set of potential images and video are filtered for those that are…

1) within a **specified distance,**
2) oriented **toward the QPoint**
3) and **visually connected**

to the Query Point.

"… an automated means for **identifying images viewing a location** through database and geo-queries greatly assists in accessing relevant images"



The **viewshed from the query point** is calculated considering intervening terrain and cover type and the height of the camera platform. Candidate imagery not within the viewshed are eliminated.

…early prototype technique developed for Red Hen Systems, Fort Collins, Colorado

(Berry)

Solid Geometry Connectivity *(Viewshed, Visual Exposure)*

Basic and Advanced Operations

www.innovativegis.com/basis/MapAnalysis/Topic15/Topic15.htm

> *Beyond Mapping III*
>
> ***Topic 15:* Deriving and Using Visual Exposure Maps**

✓ **Viewshed** marks each location that is seen from at least one "viewer" cell (binary map of seen/not seen)

✓ **Visual Exposure** generates a "density surface" indicating the number of viewer locations (count) that see each grid location (relative density)

✓ **Weighted Visual Exposure** a "density surface" of the total (sum) of the viewer weights (relative importance)

✓ **Net-Weighted Visual Exposure** a "density surface" indicating the net viewer weight (arithmetic sum) by respecting the sign of the weights (aesthetic surface where <u>pretty= positive</u> weight and <u>ugly= negative</u>)

Operation Specifications

✓ **Over** identifies the visual barrier surface (usually Elevation)

✓ **To** indicates maximum viewing distance

✓ **At** indentifies the viewer height above the terrain surface

✓ **Null** identifies an "Over" surface value indicating locations to be ignored

✓ **Opacity** uses a decay function to represent reduced visibility

✓ **Thru** an additional "blocking surface containing cells that block ✓any line of sight, such as forest canopy

✓ **Onto** map containing values reflecting the height of features above the surface map, such as smokestacks that can be seen but not blocking

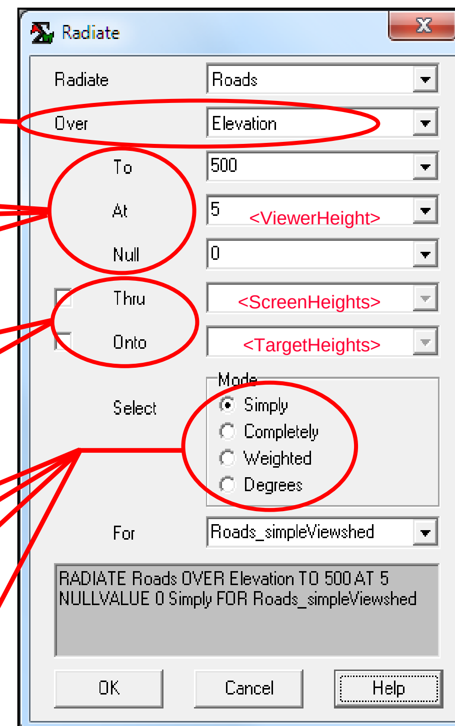
✓ **Simply** identifies all locations that are seen at least once (binary)

✓ **Completely** counts the number of "viewer" cells connected (VE)

✓ **Weighted** adds the connected viewer cell value (wVE and net wVE)

✓ **Degrees** identifies the maximum prominence angle of connected viewer cells

*<u>Radiate</u> operation in **MapCalc Learner***

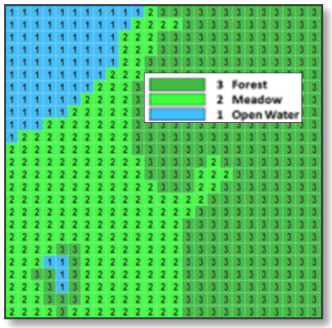


*<u>Viewshed</u> operation in **Spatial Analyst** has similar capabilities*

(Berry)

Spatial Analysis Operations *(Geographic Context)*

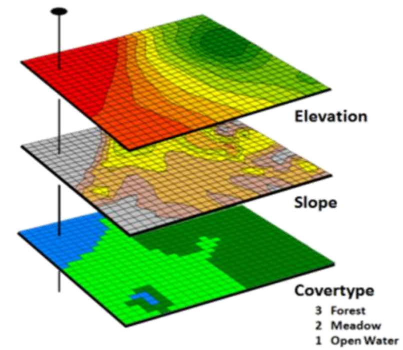
GIS as "Technical Tool" *(Where is What)* vs. "**Analytical Tool**" *(Why, So What and What if)*



Grid Layer

Spatial Analysis

Map Stack



Spatial Analysis extends the basic set of discrete map features (points, lines and polygons) to map surfaces that represent continuous geographic space as a set of contiguous grid cells, and thereby provides a mathematical/statistical framework for *analyzing* and *modeling* the

Contextual Spatial Relationships

within and among grid map layers

Map Analysis Toolbox

Mathematical Perspective:

Basic GridMath & Map Algebra *(+ - * /)*

Advanced GridMath *(Math, Trig, Logical Functions)*

Map Calculus *(Spatial Derivative, Spatial Integral)*

Map Geometry *(Euclidian Proximity, Effective Proximity, Narrowness)*

Plane Geometry Connectivity *(Optimal Path, Optimal Path Density)*

Solid Geometry Connectivity *(Viewshed, Visual Exposure)*

✓ ***Unique Map Analytics*** *(Contiguity, Size/Shape/Integrity, Masking, Profile)*

…discussion focused on these distance related groups of operations as they are least understood by the STEM disciplines — see reading references for more information on all of the operations

(Berry)